Improving Convergence Rate Of IC3

Eugene Goldberg
eu.goldberg@gmail.com

***Abstract*—*IC3*, a well-known model checker, proves a property of a transition system ξ by building a sequence of formulas $F_0,\dots,F_k$. Formula F_i, $0 \le i \le k$ over-approximates the set of states reachable in at most i transitions. The basic algorithm of *IC3* cannot guarantee that the value of k never exceeds the reachability diameter of ξ. We describe an algorithm called *IC4* that gives such a guarantee. (*IC4* stands for "*IC3* + Improved Convergence"). One can argue that the *average* convergence rate of *IC4* is better than for *IC3* as well. Improving convergence can facilitate some other variations of the basic algorithm. As an example, we describe a version of *IC4* employing *property decomposition*. The latter means replacing an original (strong) property with a conjunction of weaker properties to prove by *IC4*. We argue that addressing the convergence problem is important for making the property decomposition approach work.**

I. Introduction

IC3 is a model checker [2] that has become very popular due to its high scalability. Let ξ be a transition system and P be a safety property of ξ. *IC3* builds a sequence of formulas $F_0,\dots,F_k$ where F_i over-approximates the set of states reachable from an initial state of ξ in at most i transitions. Property P is proved when F_i becomes an inductive invariant of ξ for some $0 \le i \le k$.

One of the reasons for high performance of *IC3* is that the value of k above is typically much smaller than $Diam(\xi)$ (i.e. the reachability diameter of ξ). So, on average, *IC3* converges to an inductive invariant much faster than an RA-tool (where RA stands for "reachability analysis"). Interestingly, the *worst case* behavior of an RA-tool and *IC3* is quite different from their average behavior. Namely, *IC3* cannot guarantee that k never exceeds $Diam(\xi)$. We introduce a modification of *IC3* called *IC4* that fixes the problem above. (*IC4* stands for "*IC3* + Improved Convergence"). On one hand, *IC4* has the same worst case behavior as an RA-tool. On the other hand, the *average* convergence rate of *IC4* is arguably better than that of *IC3* as well.

The main difference between *IC4* and *IC3* is as follows. *IC3* checks if formula F_k is an inductive invariant by "pushing" the clauses of F_k to F_{k+1}. If every clause of F_k can be pushed to F_{k+1}, the former is an inductive invariant. Otherwise, there is at least one clause $C \in F_k$ that cannot be pushed to F_{k+1}. In this case, *IC3* moves on re-trying to push C to F_{k+1} when new clauses are added to F_k. In contrast to *IC3*, *IC4* applies extra effort to push C to F_{k+1}. Namely, it derives new inductive clauses to exclude states that prevent C from being pushed to F_{k+1}. This extra effort results either in successfully pushing C to F_{k+1} or in *proving* that C is "*unpushable*".

The proof of unpushability consists of finding a *reachable* state $\boldsymbol{s}$ that satisfies formula F_{k+1} and falsifies clause C. The existence of $\boldsymbol{s}$ means that F_k *cannot* be turned into an inductive invariant by adding more clauses. Thus, semantically, the difference between *IC4* and *IC3* is that the former starts building a new over-approximation F_{k+1} *only* after it proved that adding one more time frame is *mandatory*. Operationally, *IC4* and *IC3* are different in that *IC4* generates a small set of reachable states.

An appealing feature of *IC3* is its ability to generate property-specific proofs. So it seems natural to decompose a hard property P into a conjunction $P_1 \wedge \dots P_m$ of weaker properties and then generate m property-specific proofs for P_i. However, the convergence issues of *IC3* are arguably more pronounced for weak properties (see Subsection VII-B). So, to make property decomposition work, one should use *IC4* rather than *IC3* to prove properties P_i. In this paper, we describe a variation of *IC4* employing property decomposition.

At the time of writing the first version of the paper we were not aware of *QUIP*, a version of *IC3* published at [1]. We fix this omission and describe the relation between *IC4* and *QUIP* in Subsection VII-A. *QUIP* more aggressively than the basic *IC3* pushes clauses to future time frames and generates reachable states as a proof that a clause cannot be pushed. However, no relation of *QUIP*'s good performance with improvement of its convergence rate has been established either theoretically or experimentally.

The contribution of this paper is as follows. First, we show the reason why *IC3* has a poor upper bound on the convergence rate (Section III). Second, we formulate a new version of *IC3* called *IC4* (Section IV) that is meant for fixing this problem. In particular, we show that *IC4* indeed has a better upper bound than *IC3* (Section V). We also give an estimate of the number of reachable states *IC4* has to generate (Section VI). Third, we discuss arguments in favor of *IC4* (Section VII). Fourth, we describe *IC4-PD*, a version of *IC4* meant for solving hard problems by property decomposition (Section VIII).

II. A Brief Overview Of *IC3*

Let I and T be formulas[1] specifying the initial states and transition relation of a transition system ξ respectively. Let P be a formula specifying a safety property of ξ. *IC3* proves P by building a set of formulas $F_0,\dots,F_k$. Here formula F_i, $0 \le i \le k$ depends on the set of state variables of i-th time frame (denoted as S_i) and over-approximates the set of states[2]

[1] We assume that all formulas are propositional and are represented in CNF (conjunctive normal form)

[2] A state is an assignment to the set of state variables.

reachable in at most i transitions. That is every state reachable in at most i transitions is an F_i-state [3].

IC3 builds formula F_k as follows. Formula F_0 is always equal to I. Every formula F_k, $k > 0$ is originally set to P. (So $F_k \rightarrow P$ is always true because the only modification applied to F_k is adding clauses.) Then *IC3* tries to exclude every F_k-state that is a predecessor of a bad state[4] i.e. a state $\boldsymbol{s}$ that breaks $F_k \wedge T \rightarrow P'$. Here T is a short for $T(S_k, S_{k+1})$ and P', as usual, means that P depends on *next-state variables* i.e. those of S_{k+1}. Exclusion of $\boldsymbol{s}$ is done by derivation of a so-called inductive clause C falsified by $\boldsymbol{s}$. Adding C to F_k excludes $\boldsymbol{s}$ from consideration. (If $\boldsymbol{s}$ cannot be excluded, *IC3* generates a counterexample.)

One of the properties of formulas F_i maintained by *IC3* is $F_i \rightarrow F_{i+1}$. To guarantee this, *IC3* maintains two stronger properties of F_i: a) $\mathit{Clauses}(F_{i+1}) \subseteq \mathit{Clauses}(F_i)$ and b) $F_i \neq F_{i+1}$ implies that $F_i \not\equiv F_{i+1}$. That is the set of clauses of F_i contains all the clauses of F_{i+1} and the fact that F_i contains at least one clause that is not in F_{i+1} means that F_i and F_{i+1} are logically inequivalent. Since every formula F_i implies P, one cannot have more than $|P\text{-}\mathit{states}|$ different formulas $F_0, \ldots, F_k$. That is if the value of k exceeds $|P\text{-}\mathit{states}|$, there should be two formulas F_{i-1}, F_i, $i < k$ such that $F_{i-1} = F_i$. This means that F_{i-1} is an inductive invariant and property P holds.

III. Convergence Rate Of *IC3* And Clause Pushing

We will refer to the number of time frames one has to unroll before proving property P as the **convergence rate**. We will refer to the latter as $\mathit{ConvRate}(P)$. As we mentioned in Section II, an upper bound on $\mathit{ConvRate}(P)$ of the basic version of *IC3* formulated in [2] is $|P\text{-}\mathit{states}|$. Importantly, the value of $|P\text{-}\mathit{states}|$ can be much larger than $\mathit{Diam}(\xi)$ (i.e. the reachability diameter of ξ). Of course, on average, $\mathit{ConvRate}(P)$ of *IC3* is much smaller than $\mathit{Diam}(\xi)$, let alone $|P\text{-}\mathit{states}|$. However, as we argue below, a poor upper bound on $\mathit{ConvRate}(P)$ is actually *a symptom of a problem.*

Recall that formula F_k specifies an over-approximation of the set of states reachable in at most k transitions. So, it cannot exclude a state $\boldsymbol{s}$ reachable in j transitions where $j \leq k$. (That is such a state $\boldsymbol{s}$ cannot falsify F_k.) On the other hand, F_k may exclude states reachable in *at least* $k+1$ transitions or more.

Suppose *IC3* just finished constructing formula F_k. At this point $F_k \wedge T \rightarrow P'$ holds i.e. no bad state can be reached from an F_k-state in one transition. After constructing F_k, *IC3* invokes a procedure for pushing clauses from F_k to F_{k+1}. In particular, this procedure checks for every clause C of F_k if implication $F_k \wedge T \rightarrow C'$ holds. We will refer to this implication as the **pushing condition**. If the pushing condition holds for clause C, it can be pushed from F_k to F_{k+1}. If the pushing condition holds for every clause[5] of F_k, then $F_k \wedge T \rightarrow F'_k$ and F_k is an inductive invariant.

Suppose that the pushing condition does not hold for a clause C of F_k. Below, we describe two different reasons for the pushing condition to be broken. *IC3* does not try to identify which of the reasons takes place. This feature of *IC3* is the cause of its poor upper bound on $\mathit{ConvRate}(P)$. Moreover, intuitively, this feature should affect the *average* value of $\mathit{ConvRate}(P)$ as well.

The first reason for breaking the pushing condition is that clause C excludes a state $\boldsymbol{s}$ that is *reachable* in $(k+1)$-th time frame from an initial state. In this case, formula F_k *cannot* be turned into an inductive invariant by adding more clauses. In particular, the broken pushing condition cannot be fixed for C. The second reason for breaking the pushing condition is that clause C excludes a state $\boldsymbol{s}$ that is *unreachable* in $(k+1)$-th time frame from an initial state. In this case, every F_k-state $\boldsymbol{q}$ that is a predecessor of $\boldsymbol{s}$ can be excluded by deriving a clause falsified by $\boldsymbol{q}$. So in this case, the broken pushing condition *can* be fixed. In particular, by fixing broken pushing conditions for F_k one may turn the latter into an inductive invariant.

IV. Introducing *IC4*

A. *A high-level view of IC4*

We will refer the version of *IC3* with a better convergence rate described in this paper as ***IC4***. The main difference between *IC3* and *IC4* is that the latter makes an extra effort in pushing clauses to later time frames. This new feature of *IC4* is implemented in a procedure called *NewPush* (see Figure 1). It is invoked after *IC4* has built F_k where the predecessors of bad states are excluded i.e. as soon as $F_k \wedge T \rightarrow P'$ holds. For every clause C of F_k, *NewPush* checks the pushing condition (see Section III). If this condition is broken, *NewPush* tries to fix it or proves that it cannot be fixed and hence C is "unpushable".

Depending on the clause-pushing effort, one can identify three different versions of *IC4*: minimal, maximal and heuristic. The *minimal IC4* stops fixing pushing conditions as soon as *NewPush* finds a clause of F_k that cannot be pushed. After that the minimal *IC4* switches into the "*IC3* mode" where the pushing conditions are not fixed for the remaining clauses of F_k. The *maximal IC4* tries to fix the pushing condition for every inductive clause of F_k. That is if a clause $C \in F_k$ cannot be pushed to F_{k+1}, the maximal *IC4* tries to fix the pushing condition (regardless of how many unpushable clauses of F_k has been already identified). Moreover, if an inductive clause C is added to F_i, $i < k$, the maximal *IC4* try to fix the pushing condition for C if it cannot be immediately pushed to F_{i+1}.

A *heuristic IC4* uses a heuristic to stay between minimal and maximal *IC4* in terms of the clause-pushing effort. In this paper, we describe the minimal *IC4* unless otherwise stated. So, when we just say *IC4* we mean the minimal version of it.

[3]Given a formula $H(S)$, a state $\boldsymbol{s}$ is said to be an H-state if $H(\boldsymbol{s}) = 1$.

[4]Given a property P, a $\overline{P}$-state is called a bad state.

[5]In reality, since both F_k and F_{k+1} contain the clauses of P, only the inductive clauses of F_k added to strengthen P are checked for the pushing condition.

// $\mathbb{F}_k = \{F_0, \ldots, F_k\}$;
//
$NewPush(I, T, P, \mathbb{F}_k)\{$
1 $NewClauses := true$;
2 $F_{k+1} := P$
3 while ($NewClauses$) {
4 $NewClauses := false$;
5 foreach $C \in (F_k \setminus P)$ {
6 if ($C \in (F_{k+1} \setminus P)$) continue;
7 $\boldsymbol{s}$:= $SAT(F_k \wedge T \wedge \overline{C'})$;
8 if ($\boldsymbol{s} = nil$) {
9 $F_{k+1} := F_{k+1} \cup \{C\}$
10 continue; }
11 $(\mathbb{F}_k, \boldsymbol{t}) := ExclState(\boldsymbol{s}, I, T, P, \mathbb{F}_k)$;
12 if ($\boldsymbol{t} \neq nil$) return($C, \boldsymbol{t}$);
13 $NewClauses := true$}}
14 return(nil, nil); }

Fig. 1. The $NewPush$ procedure

B. Description of NewPush

The pseudo-code of *NewPush* is given in Fig. 1. At this point *IC4* has finished generation of F_k. In particular, no bad state can be reached from an F_k-state in one transition. *NewPush* tries to push every inductive clause of F_k to F_{k+1}. If a clause $C \in F_k$ is unpushable, *NewPush* returns C and a trace $\boldsymbol{t}$ leading to a state falsified by clause C. Trace $\boldsymbol{t}$ proves the unpushability of C and hence the fact that F_k cannot be turned into an inductive invariant by adding more clauses. If every clause of F_k can be pushed to F_{k+1}, then F_k is an inductive invariant and *NewPush* returns (*nil*, *nil*) instead of clause C and trace $\boldsymbol{t}$.

NewPush consists of two nested loops. A new iteration of the outer loop (lines 3-13) starts if variable *NewClauses* equals *true*. The value of this variable is set in the inner loop (lines 5-13) depending on whether new clauses are added to F_k. In every iteration of the inner loop, *NewPush* checks the pushing condition (line 7) for an inductive clause of F_k that is not in F_{k+1}. If it holds, then C is pushed to F_{k+1}.

If the pushing condition fails, an F_{k+1}-state $\boldsymbol{s}$ is generated that falsifies clause C. Then *NewPush* tries to check if $\boldsymbol{s}$ is reachable exactly as *IC3* does this when looking for a counterexample. The only difference is that $\boldsymbol{s}$ is a good state[6]. As we mentioned above, if $\boldsymbol{s}$ is reachable by a trace $\boldsymbol{t}$, *NewPush* terminates returning C and $\boldsymbol{t}$. Otherwise, it sets variable *NewClauses* to *true* and starts a new iteration of the inner loop.

V. Better Convergence Rate of *IC4*

As we mentioned in Section II, an upper bound on $ConvRate(P)$ is $|P\text{-}states|$. Below, we show that using procedure *NewPush* described in Section IV brings the upper bound on $ConvRate(P)$ for *IC4* down to $Diam(\xi)$. (Note that if property P holds, $Diam(\xi) \leq |P\text{-}states|$.)

Let F_k be a formula for which *NewPush* is called when $k \geq Diam(\xi)$. At this point $F_k \wedge T \rightarrow P'$ holds. Let $\boldsymbol{s}$ be a state breaking the pushing condition for a clause C of F_k. That is $\boldsymbol{s}$ falsifies C (and hence it is not an F_k-state) but is reachable from an F_k-state in one transition.

Recall that F_k is an over-approximation of the set of states that can be reached in at most k-transitions. Since $\boldsymbol{s}$ falsifies F_k, reaching it from an initial state of ξ requires at least $k+1$ transitions. However, this is impossible since $k+1 > Diam(\xi)$ and hence state $\boldsymbol{s}$ is unreachable. This means that every F_k-state that is a predecessor of $\boldsymbol{s}$ can be excluded by an inductive clause added to F_k. So eventually, *NewPush* will fix the pushing condition for C. After fixing all broken pushing conditions for clauses of F_k, *NewPush* will turn F_k into an inductive invariant.

VI. Number Of Reachable States To Generate

The number of generated reachable states depends on which of the three versions of *IC4* is considered (see Subsection IV-A). Let k denote the maximal number of time frames unfolded by *IC4*. In the case of the minimal *IC4*, the upper bound on the number of reachable states for proving property P is equal[7] to $k * (k+1)/2$. For the maximal *IC4*, the upper bound is $k * |Unpush(F)|$ where $F = F_1 \cup \cdots \cup F_k$ and $Unpush(F)$ is the subset of F consisting of unpushable clauses. Indeed, an inductive clause $C \in F_i$ is proved unpushable only once. This proof consists of a trace to a state falsified by F_i. The length of this trace is equal to i and hence bounded by k. The upper bound for the maximal *IC4* above is loose because one assumes that

- the length of every trace proving unpushability equals k
- two (or more) clauses cannot be proved unpushable by the same reachable state.

Re-using reachable states can dramatically reduce the total number of reachable states one needs to generate. For instance, for the minimal *IC4*, this number can drop as low as k. For the maximal *IC4*, the total number of reachable states can go as low as $m + k$ where m is the total number of reachable states generated to prove the unpushability of clauses of $Unpush(F)$.

VII. A Few Arguments In Favor Of *IC4*

In this section, we give some arguments in favor of *IC4*. The main argument is given in Subsection VII-A where we relate *IC4* with a model checker called *QUIP*. The latter was introduced[8] in [1] in 2015. In Subsections VII-B and VII-C, we describe a few potential advantages of *IC4* that were not discussed in [1] (in terms of *QUIP*).

[6] Recall that at this point of the algorithm, no bad state can be reached from an F_k-state in one transition.

[7] For every formula F_i, $i = 1, \ldots, k$, *IC4* generates one reachable state $\boldsymbol{s}$ falsifying a clause of F_i. To reach $\boldsymbol{s}$, one needs to generate a trace of i states. So the number of reachable states generated for F_i is equal to i. The total number of reachable states is equal to $1 + 2 + \ldots + k$.

[8] As we mentioned in the introduction, at the time of writing the first version of our paper we were not aware of *QUIP*.

A. IC4 and QUIP

As we mentioned in the introduction, *QUIP* makes an extra effort to push clauses to future time frames. To show that a clause cannot be pushed, *QUIP* generates a reachable state. Although the premise of *QUIP* is that the strategy above may lead to a faster generation of an inductive invariant, this claim has not been justified theoretically. The advantage of *QUIP* over *IC3* is shown in [1] in terms of better run times and a greater number of solved problems. So, no *direct* experimental data is provided on whether *QUIP* has a better convergence rate than *IC3*. (As mentioned in [1] and in the first version of our paper, having at one's disposal reachable states facilitates construction of better inductive clauses[9]. So one cannot totally discard the possibility that the performance of *QUIP* is mainly influenced by this "side effect".) Nevertheless, great experimental results of *QUIP* is an encouraging sign.

B. Proving weak properties

In this subsection, we argue that *IC4* should have more robust performance than *IC3* on weak properties. Let F_i be an over-approximation of the set of states reachable in at most i transitions and P be the property to prove. As we mentioned earlier, there are two conditions one needs to satisfy to turn F_i into an inductive invariant: $F_i \wedge T \rightarrow P'$ and $F_i \wedge T \rightarrow F'_i$. We will refer to a state $\boldsymbol{s}$ breaking the first condition (respectively second condition) as a state of the first kind (respectively second kind). Only states of the first kind (i.e. F_i-states from which there is a transition to a bad state) are *explicitly* excluded by *IC3*. States of the second kind are excluded *implicitly* via generalization of inductive clauses. On the other hand, *IC4* excludes states of both kinds explicitly and implicitly (via generalization of inductive clauses).

First, assume that P is a *strong* property meaning that there is a lot of bad states. Then by excluding states of the first kind coupled with generalization of inductive clauses, *IC3* also excludes many states of the second kind. Now assume that P is a *weak* property that has, say, only one bad state. Let us also assume that excluding states reaching this bad state is easy. Intuitively, in this case, *IC3* is less effective in excluding the states of the second kind (because their exclusion is just a *side effect* of excluding states of the first kind). On the other hand, *IC4* does not have this problem and so arguably should have a more robust behavior than *IC3* when proving weak properties.

C. Test generation

Formal verification of *some* properties of transition system ξ does not guarantee that the latter is correct[10]. In this case, testing is employed to get more confidence in correctness of ξ. Traces generated by *IC4* can be used as tests in two scenarios. First, one can check that reachable states found by *IC4* satisfy the properties that formal verification tools failed to prove. Second, one can just inspect the states visited by ξ and the outputs produced in those states to check if they satisfy some (formal or informal) criteria of correctness.

[9]By avoiding the exclusion of known reachable states, one increases the chance for an inductive clause to be a part of an inductive invariant.

[10]Moreover, ξ can be incorrect even if a supposedly complete set of properties $P_1, \ldots, P_n$ is proved true [4], [3]. For instance, the designer may "misdefine" a property and so instead of verifying the right property P'_i (that does not hold) a formal tool checks a *weaker* property P_i (that holds).

IC4-PD$(I, T, P)\{$
$Inv := \emptyset$
while (*true*) $\{$
  $\boldsymbol{s} := CheckSat(Inv \wedge \overline{P})$
  if $(\boldsymbol{s} = nil)$ return(Inv, nil)
  $Q := FormProp(\boldsymbol{s})$
  $(J, Cex) := IC4^*(I, T, P, Inv, Q)$
  if $(Cex \neq nil)$ return(nil, Cex)
  if $(J = Q)$
    $J := Strengthen(I, T, Inv, J)$
  $Inv := Inv \wedge J\ \}\ \}$

Fig. 2. The *IC4-PD* procedure

VIII. Introducing *IC4-PD*

In this section, we present *IC4-PD*, a version of *IC4* employing property decomposition. In Subsection VIII-A, we describe two obstacles one has to overcome to make property decomposition work. Subsection VIII-B introduces a straightforward implementation of *IC4-PD*.

A. Property decomposition: two obstacles to overcome

As we mentioned in the introduction, an appealing feature of *IC3* is its ability to generate property-specific proofs. Let P be a hard property to prove. Let P be represented as $P_1 \wedge \cdots \wedge P_k$ (i.e. P is decomposed into k weaker properties). Let J_k be an inductive invariant for property P_k. Then $J_1 \wedge \cdots \wedge J_k$ is an inductive invariant for property P. So one can prove P via finding property-specific proofs J_i, $i = 1, \ldots, k$.

To make the idea of property decomposition work one has to overcome at least two obstacles. The first obstacle[11] is that the search space one has to examine to prove P_i is, in general, not a subset[12] of the search space for P. In [5], we show that this issue can be addressed by using the machinery of local proofs[13].

The second obstacle is as follows. As we argued in Subsection VII-B, weak properties are more likely to expose the convergence rate problem of *IC3*. For that reason, replacing a strong property P with weaker properties P_i may actually lead to performance degradation if properties P_i are proved by *IC3*. On the other hand, *IC4* should be more robust when solving weak properties. So one can address the second obstacle by using *IC4* (rather than *IC3*) to prove properties P_i.

[11]This obstacle is of a general nature and is not caused by using *IC3*.

[12]The reason is that when proving P_i one may need to consider traces that contain two and more $\overline{P}$-states. These traces break property P without breaking property P_i.

[13]To prove that P_i holds *globally* one needs to show that no trace of P_i-states reaches a $\overline{P}_i$-state. Proving P_i *locally* means showing that no trace of P-states (rather than P_i-states) reaches a $\overline{P}_i$-state. As we show in [5], if P is false, there is property P_i that breaks both globally and locally. So if every P_i holds locally, then it does globally too and P is true.

B. Description of IC4-PD

The pseudocode of *IC4-PD* is shown in Fig. 2. *IC4-PD* accepts formulas I, T, P specifying the initial states, the transition relation and the property to prove respectively. *IC4-PD* returns either an inductive invariant Inv or a counterexample Cex. Computation is performed in a *while* loop. First, *IC4-PD* checks if there is a $\overline{P}$-state $\boldsymbol{s}$ breaking $Inv \rightarrow P$ (line 3). If not, then *Inv* is an inductive invariant proving P (line 4). Otherwise, *IC4-PD* forms a new property Q to prove (line 5). Q consists of one clause, namely, the longest clause falsified by $\boldsymbol{s}$. So, the latter is the only $\overline{Q}$-state.

Then *IC4-PD* calls *IC4**, a version of *IC4* that proves Q *locally*[14] with respect to the target property P (see Subsection VIII-A). That is *IC4** checks is there is a trace of P-states (rather than Q-states) leading to the $\overline{Q}$-state. If not, then Q holds locally. *IC4** uses the current Inv as a constraint[15]. Namely, *IC4** looks for a formula J satisfying $Inv \wedge J \wedge T \rightarrow J'$ (rather than $J \wedge T \rightarrow J'$).

If *IC4** finds a counterexample Cex, then Q and hence P fail (line 7). Otherwise, *IC4** returns an inductive invariant J. If Q is itself an inductive property (and so $J = Q$), *IC4* tries to strengthen J like an inductive clause is strengthened by *IC3* (line 9). This is done to avoid enumerating $\overline{P}$-states one by one if many properties Q turn out to be inductive. If J is already strengthened (and so $J \neq Q$), then Inv is replaced with $Inv \wedge J$ and a new iteration begins.

References

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[14] Proving Q locally addresses the first obstacle mentioned in Subsection VIII-A. The second obstacle is addressed by using *IC4* instead of *IC3*.

[15] It is safe to do because all reachable states satisfy Inv.