

# **Complete Test Sets And Their Approximations**

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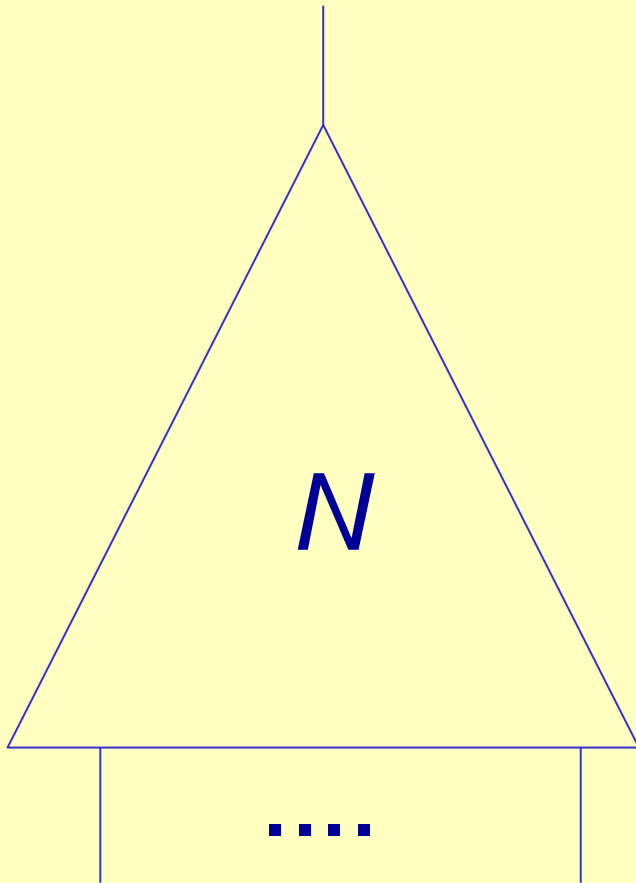
***Austin, TX, USA***

***October 30 – November 2, 2018***

# Outline

- Introduction
- Complete Test Sets (CTSs)
- Experiments and conclusions

# The Problem

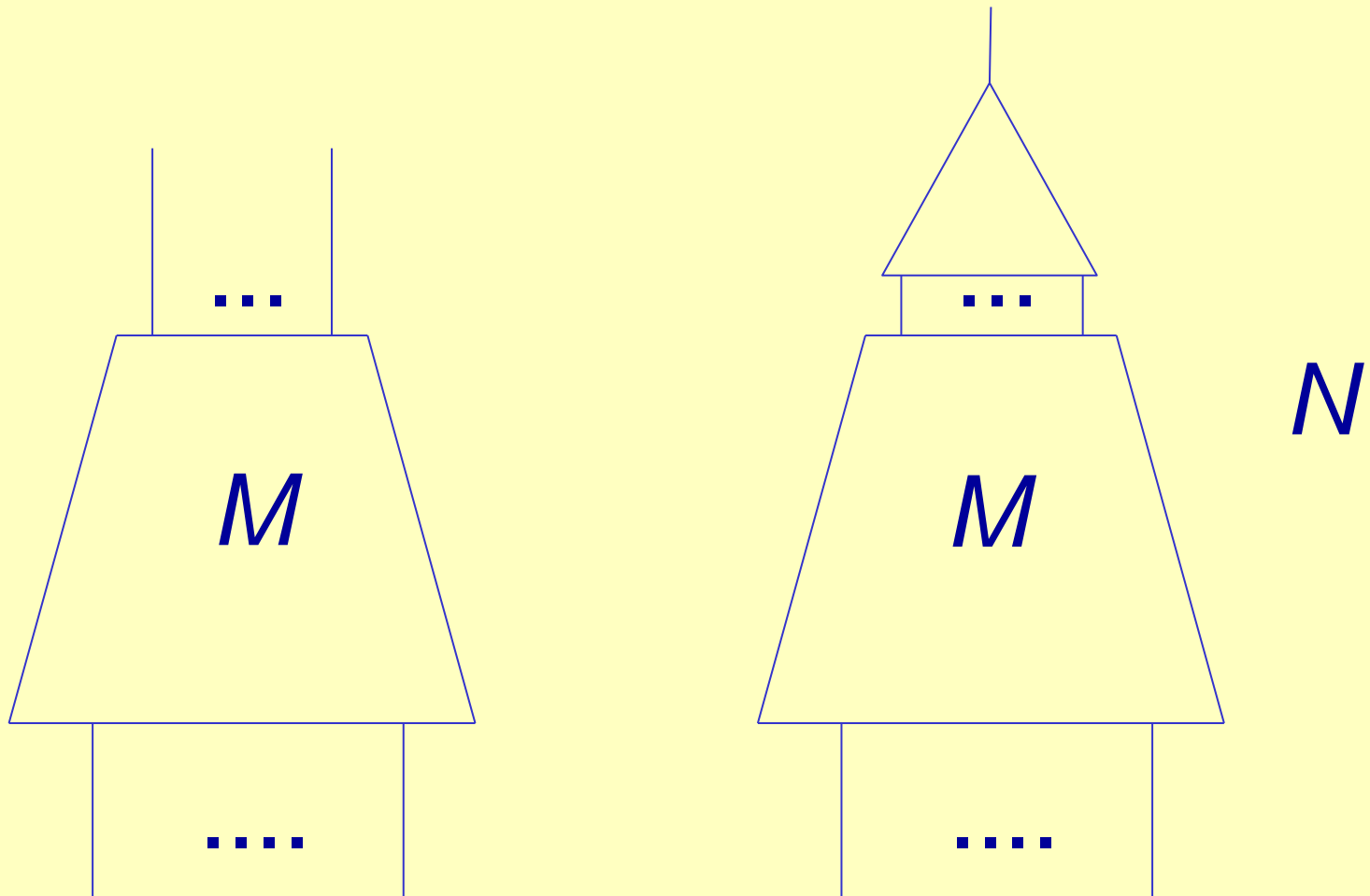


The problem we consider:  
Check if  $N \equiv 0$

$N \equiv 0$  denotes the fact that  
 $N$  outputs 0 for every input

We want to prove  
 $N \equiv 0$  by testing

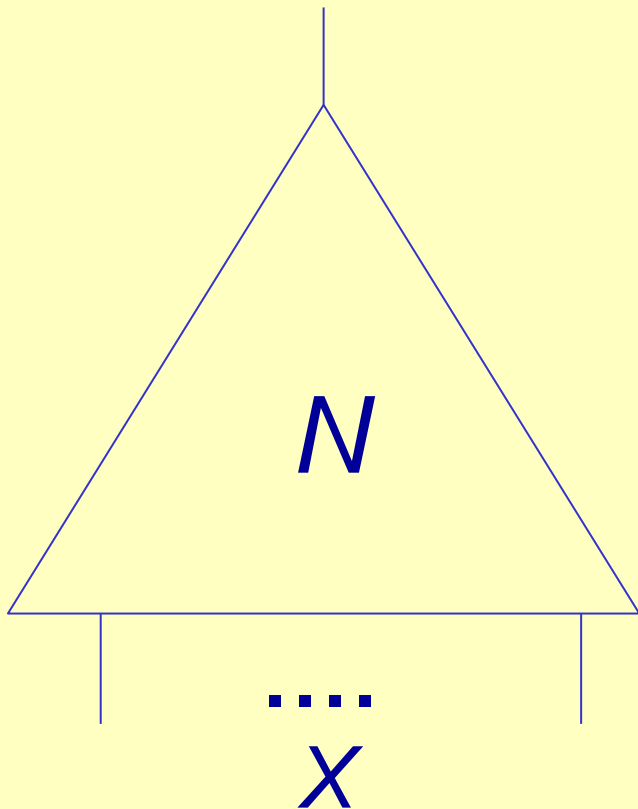
# The Context



# Complete Test Set (CTS)

Test  $x$  is an assignment to  $X$

Test set  $T = \{x_1, \dots, x_m\}$ ,



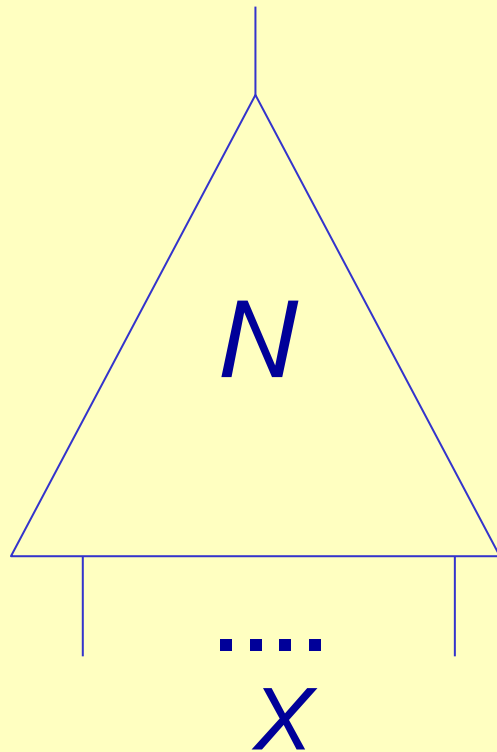
$T$  is a **CTS** if

$$N(T) = 0 \Rightarrow N \equiv 0$$

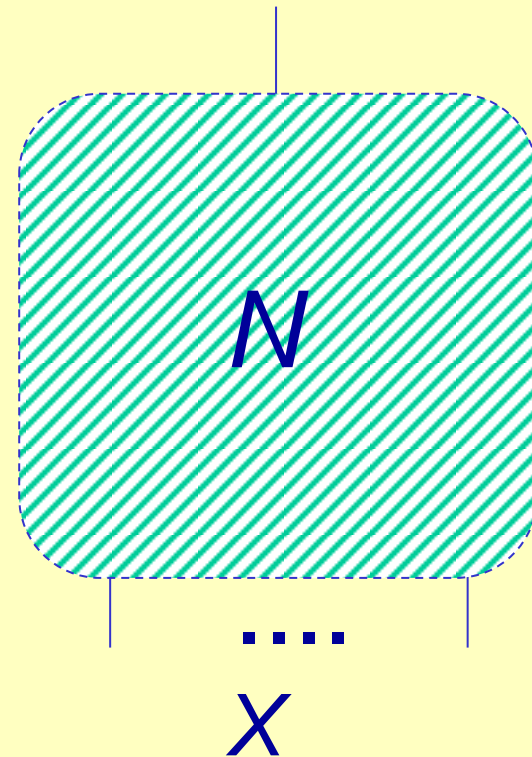
$T$  is a **trivial CTS**

if  $|T| = 2^{|X|}$

# Black/White Box Testing



$$|T_{CTS}| \leq 2^{|X|}$$



$$|T_{CTS}| = 2^{|X|}$$

# Testing as Structural Derivation

$N \equiv 0$  is a **semantic property** of  $N$ :

$(N \equiv 0) \wedge (N^* \equiv N)$  implies  $N^* \equiv 0$

A non-trivial CTS is a **structural property** of  $N$ :

$T$  is a CTS for  $N$  and  $N^* \equiv N \not\Rightarrow$

$T$  is a CTS for  $N^*$

**Testing:** Make a *semantic* derivation ( $N \equiv 0$ ) by proving a *structural* property (non-trivial CTS)

# Some Applications Exploiting Reusability of Tests

Let  $\xi$  be a property of  $M$ . Formal proof of  $\xi$  is **hard to reuse**.

Let  $N \equiv 0 \Leftrightarrow \xi$  holds

Let  $T$  be generated to test  $N$

Set  $T$  **can be reused**

- to check other properties of  $M$
- to check input/output behavior of  $M$
- to check  $\xi$  after  $M$  is modified



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# Stable Set of Assignments (SSA)

Given CNF formula  $G(W)$ ,  $P = \{q_1, \dots, q_m\}$  is an **SSA**

- $\forall q_i \in P, G(q_i) = 0$
- $P$  is closed w.r.t. to a neighborhood relation

$G$  is **unsatisfiable** iff it has an SSA

Trivial SSA: all  $2^{|W|}$  assignments

Non-trivial SSA is a **structural property**:

$P$  is an SSA for  $G$  and  $G^* \equiv G \not\Rightarrow$

$P$  is an SSA for  $G^*$

# Example of SSA

$$G = C_1 \wedge \dots \wedge C_4,$$

$$C_1 = W_1 \vee W_2 \vee W_3,$$

$$C_2 = \sim W_1,$$

$$C_3 = \sim W_2,$$

$$C_4 = \sim W_3$$

$\mathbf{q}_1 = (w_1=0, w_2=0, w_3=0)$  falsifies  $C_1$

$$\text{Nbhd}(\mathbf{q}_1, C_1) = \{\mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4\}$$

$$\mathbf{q}_2 = (w_1=1, w_2=0, w_3=0),$$

$$\mathbf{q}_3 = (w_1=0, w_2=1, w_3=0),$$

$$\mathbf{q}_4 = (w_1=0, w_2=0, w_3=1),$$

# Example of SSA (continued)

$$C_1 = W_1 \vee W_2 \vee W_3, \quad C_2 = \sim W_1, \quad C_3 = \sim W_2, \quad C_4 = \sim W_3$$

$$P = \{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4\},$$

$$\mathbf{q}_1 = (0 \ 0 \ 0), \quad \mathbf{q}_2 = (1 \ 0 \ 0), \quad \mathbf{q}_3 = (0 \ 1 \ 0), \quad \mathbf{q}_4 = (0 \ 0 \ 1)$$

$$\text{Nbhd}(\mathbf{q}_1, C_1) = \{\mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4\}$$

$$\text{Nbhd}(\mathbf{q}_2, C_2) = \{\mathbf{q}_1\}$$

$$\text{Nbhd}(\mathbf{q}_3, C_3) = \{\mathbf{q}_1\},$$

$$\text{Nbhd}(\mathbf{q}_4, C_4) = \{\mathbf{q}_1\}$$

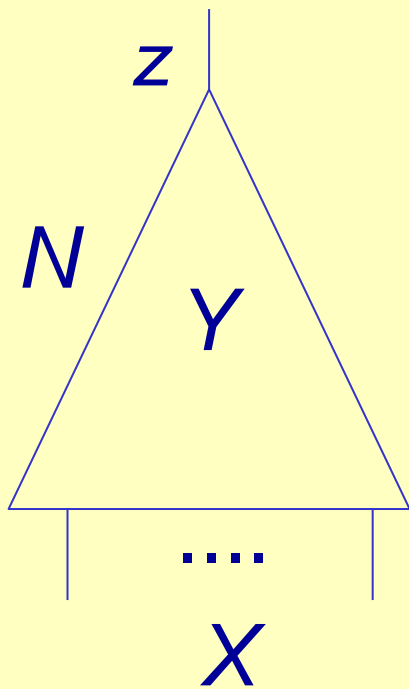
$P$  is **closed**:  $\forall \mathbf{q}_k \in P, \exists C_j \in G$

s.t.  $C_j(\mathbf{q}_k) = 0$  and  $\text{Nbhd}(\mathbf{q}_k, C_j) \subseteq P$

$P$  is an **SSA** for  $G = C_1 \wedge \dots \wedge C_4$

# Building Complete Test Set

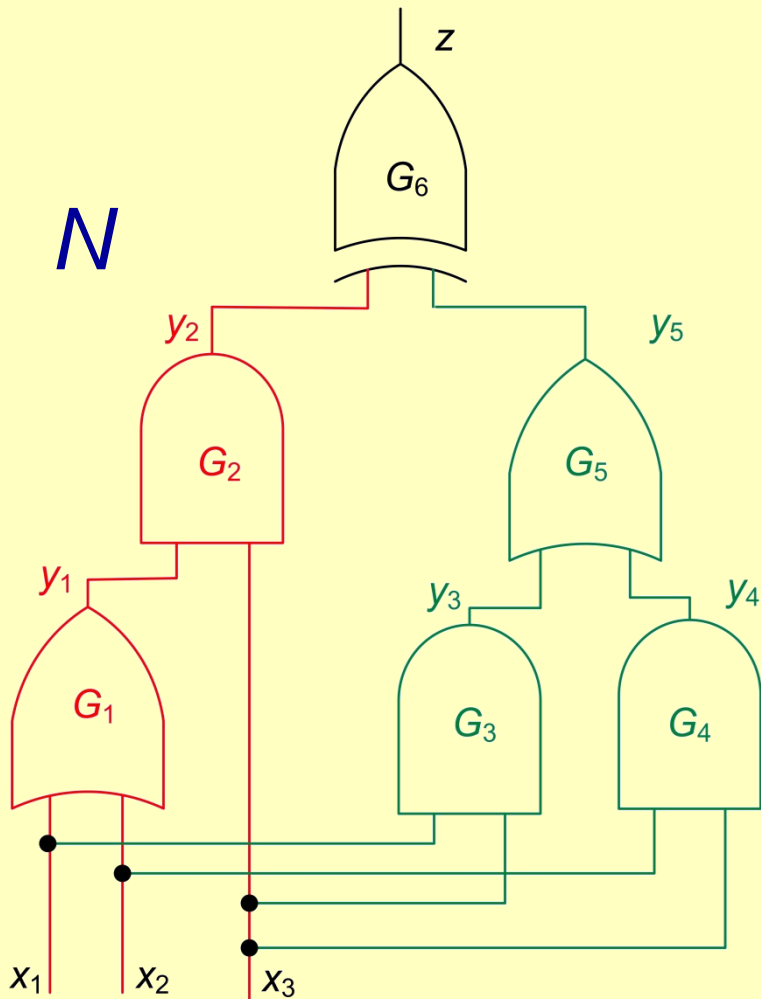
Let  $F_N(X, Y, z)$  be CNF specifying  $N$   
 $N \equiv 0 \Leftrightarrow F_N \wedge z \equiv 0$



1. Build SSA  $\{\mathbf{q}_1, \dots, \mathbf{q}_m\}$  for  $F_N \wedge z$
2. Form  $T = \{\mathbf{x}_1, \dots, \mathbf{x}_m\}$ ,  $\mathbf{x}_i = \text{proj}(\mathbf{q}_i, X)$ ,  $i=1, \dots, m$
3. Remove duplicates from  $T$

$T$  is a **CTS** for  $N$

# Example of CTS



$$(x_1 \vee x_2) \wedge x_3 \equiv (x_1 \wedge x_3) \vee (x_2 \wedge x_3)$$

$F_N \wedge z$  has SSA  $P$  of 21 assignments to  $X \cup Y \cup \{z\}$

where  $X = \{x_1, x_2, x_3\}$ ,  $Y = \{y_1, \dots, y_5\}$

$P$  has 5 different assignments to  $X \Rightarrow$  **CTS of 5 tests**

# CTSs Are Too Large

Even non-trivial CTSs are too large  $\Rightarrow$   
Approximate CTS (denoted as  $\text{CTS}^{\text{aprx}}$ )

Build  $T$  for a projection of  $N$  on  $V \subset X \cup Y \cup \{z\}$

1. Generate  $G(V)$  implied by  $F_N \wedge z$
2. Build SSA  $P$  for  $G$
3. Extract test set  $T$  from  $P$

Proving  $F_N \wedge z \equiv 0$  in two steps.

- **Semantic step:**  $F_N \wedge z \Rightarrow G$
- **Structural step:** SSA  $P$  for  $G$

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# Testing Misdefined Properties

- Property  $\xi$  of sequential circuit  $M$  is **misdefined**
  - $\xi$  holds while the correct property  $\xi^*$  does not
- False positives are hard to deal with
  - Propping up formal verification by testing (assuming that  $\xi$  and  $\xi^*$  are close)
1. Form  $N_k$ , where  $N_k \equiv 0 \Leftrightarrow \xi$  holds for  $k$  transitions
  2. Build CTS<sup>aprx</sup>  $T$  for a projection of  $N_k$ .
  3. Run  $T$  to test  $M$  for  $k$  transitions

# Description of Experiment

- HWMCC-10 benchmarks are used
- The original (true) property  $\xi$  is misdefined
- The “correct” property  $\xi^*$  fails in  $k$  transitions

Let  $N_k$  and  $N_k^*$  specify  $\xi$  and  $\xi^*$  for  $k$  transitions

1. Generate a CTS<sup>aprx</sup>  $T$  to prove  $N_k \equiv 0$
2. Run  $T$  to break  $N_k^* \equiv 0$
3. Compare  $T$  with random and coverage tests

# Some Results

name	#time frames	#invars	#gates $\times 10^3$	cov. metric		random		CTS <sup>aprx</sup>	
				#tests	time (s)	#tests	time (s)	#tests	time (s)
bobco..	19	38	1.6	740	0.4	$1.0 \cdot 10^7$	294	3,339	1.1
cmugig..	4	88	4.3	2,150	6.3	$1.4 \cdot 10^6$	158	923	3.7
eijks256	39	117	18	8,976	70	$4.5 \cdot 10^6$	5,000	183	31
kenopp1	3	129	1.7	1,202	0.5	$10^8$	695	1,344	0.4
nusmv-guidan..	6	504	10	7,922	27	$2.1 \cdot 10^7$	5,000	378	2.3
nusmvt-casp2	7	1,029	19	11,510	82	$4.5 \cdot 10^7$	5,000	3,549	53
cmupe-riodic	34	1,220	51	30,999	760	$9.5 \cdot 10^6$	5,000	5,611	240
pj2002	4	4,054	137	61,113	3,868	$0.6 \cdot 10^6$	5,000	161	7.9

# Conclusions

- White-box testing  $\Rightarrow$  non-trivial CTS
- Even a non-trivial CTS is usually impractical
- Build  $\text{CTS}^{\text{aprx}}$ , approximation of CTS
- $\text{CTS}^{\text{aprx}}$  can be computed efficiently
- $\text{CTS}^{\text{aprx}}$  preserves high quality of CTS
- Our approach has numerous applications